THE EFFECT OF IRREGULAR TOOTH PITCH ON STABILITY OF MILLING

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INTRODUCTION

Recently, several authors have reported the favourable effect of irregular pitch of milling cutter teeth on the stability of the cutting process. This effect is caused by the influence of the relation between speed, chatter frequency and tooth pitch on the regeneration of chatter.

The effect of irregular tooth pitch is analysed in this paper. Firstly, the theory of chatter of Tlustý and Poláček is extended by the inclusion of the geometric condition for the phase shift undulations in subsequent cuts and secondly, the derived method is applied to the case of cutters with irregular pitch.

THE INFLUENCE ON STABILITY OF THE GEOMETRIC CONDITION ON THE PHASE SHIFT OF SUBSEQUENT CUTS

The theory of chatter as developed by Tlustý and Poláček is used herein as the basis and reference is made to the paper "A Method of Analysis of Machine Tool Stability" by Tlustý,[1] where the original theory has been extended by the inclusion of a possible phase shift ρ between the chip thickness variation and the cutting force variation. In this theory it is assumed that the phase shift ψ of two subsequent surface undulations adjusts itself to a value which is compatible with the conditions corresponding to the maximum energy of self-excitation.

In ref. 1 a comment has been made that in turning or milling a geometric condition for the phase shift ψ existed (see equation (7) and Fig. 3 in ref. 1).

This equation (equation (7) in ref. 1) is:

$$\psi = \frac{\omega \; l}{v} \tag{1}$$

where ω is the circular frequency of chatter, l is the pitch of the teeth and v is the cutting speed.

In what follows, the influence of this geometric condition on stability is investigated and a graphical method is developed. The milling process has been chosen for investigation since in this process, because of the comparatively short distance between two subsequent cuts, the effect is very strong. Three simplifications (see Fig. 1) are assumed:

I. The motion of the tools is rectilinear and the width and depth of cut are constant.
II. The tools vibrate simultaneously with respect to the workpiece and they have equal amplitudes of vibration.
III. Regular tooth pitch is assumed.

The following equations from ref. 1 (denoted therein (1), (10), (2), (8) respectively) are used:

$$P = -R(Y - Y_0) \tag{2}$$

$$Y = Y_0 e^{-j\psi} \tag{3}$$

$$Y = P\,\Phi(\omega) = PFe^{-j\psi} = P(G + jH) \tag{4}$$

$$R = r\,e^{j\rho} \tag{5}$$

where (2) expresses the dependence of the cutting force on the chip thickness, (3) is the condition for the limit of stability, (4) is the response of the vibratory system of the machine

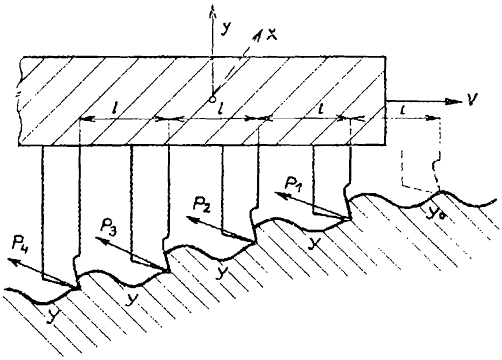

FIG. 1.

to the variable cutting force, Φ signifies the cross-receptance of the system and (5) expresses the complex coupling coefficient with ρ as the phase shift between the variation of chip thickness and of the force.

Following the assumptions I, II, III the forces acting on the individual tools may be derived from (2) and (3) as (see Fig. 1):

$$P_1 = -R_0(Y - Ye^{j\psi})$$

$$P_2 = -R_0(Y - Ye^{j\psi})$$

$$P_3 = -R_0(Y - Ye^{j\psi}) \text{ etc.}$$

Forces P_i are in phase:

$$P = \sum_{1}^{n} P_i = -n\,R_0(Y - Y\,e^{j\psi}) = -R\,Y(1 - e^{j\psi}) \tag{6}$$

where n is the number of tools cutting simultaneously and

$$R = n\,R_0 \tag{6a}$$

Combining equations (4), (5), (6), the following expression is obtained:

$$(e^{j\psi} - 1) = \frac{1}{r\,F}\,e^{j(\psi-\rho)} \tag{7}$$

and
$$2\left[\sin\frac{\psi}{2}\right]e^{j(\psi/2+\pi/2)}=\frac{1}{r\,F}\,e^{j(\psi-\rho)} \tag{7a}$$

Equation (7a) may be resolved into the equations between the arguments and the moduli separately:

$$\frac{\psi}{2}+\frac{\pi}{2}=\psi-\rho \tag{8}$$

$$r=\frac{1}{2\,F\sin\psi/2}=-\frac{1}{2\,(G\cos\rho-H\sin\rho)} \tag{9}$$

and if
$$\rho=0\ldots r=-\frac{1}{2G} \tag{9a}$$

Equation (9) is the equation for the value of the coupling coefficient on the limit of stability as derived by Tlustý in ref. 1.

Equation (8) is very important. All its terms are functions of the frequency ω. The phase shift ϕ between the exciting force and the vibration is one of the parameters of the cross-receptance of the system and is normally obtained as a function of ω by measurement when the system is excited by a vibrator. The phase shift ρ between the chip thickness and the force as a function of frequency ω has been established by specially arranged experiments by several authors (see e.g. ref. 3). Usually, as in the method of Tlustý and Poláček, this phase shift is neglected and it is assumed that $\rho=\sigma$. In this case, equation (8) transforms into

$$\frac{\psi}{2}+\frac{\pi}{2}=\psi \tag{8a}$$

Equation (8) or (8a) determines the phase shift ψ between undulations of subsequent cuts for cases at the limit of stability. However, this phase shift ψ is also independently determined by the geometric condition (1). Hence, the condition that both equations (1) and (8) (or (8a) respectively) be satisfied, determines the frequency of chatter, i.e. it determines which of the limit cases of equation (9) are compatible with the geometric condition (1). This is the distinction to the solution of the "critical" limit case derived in ref. 1, where from all limit cases one has been selected, in which the value of r (or the value of chip width l) is a minimum. With the geometric condition imposed the "critical" case is not free to occur.

The reason the geometric condition was not used in (1) was that the author was primarily concerned with the design of the machine tool. In various operations performed on a machine tool, the values in the geometric condition vary, so that a case may always be encountered which will be a "critical" case. Herein, however, the investigation is concerned with the question of how, in particular cases, with particular values in (1) an improvement of stability by the inclusion of (1) can be attained.

Simultaneous solution of (1) and (8) may be best carried out graphically, see Fig. 2. It should be realised that equation (1), if particular values of the speed v and of the pitch l are chosen, expresses ψ as a linear function of ω, periodic in the interval $0\leqq\psi\leqq 2\pi$. Then the left-hand side of (8) is a linear function periodic in the interval $\pi/2\leqq\psi/2+\pi/2\leqq 3/2\,\pi$ as shown in (c) in Fig. 2. In (a) the values of $\psi\,(\omega)$ are plotted as they have been measured on a particular machine tool and in (b) values of $\rho\,(\omega)$ according to ref. 3, are given. The curve of the difference $\psi-\rho$ is plotted also in (c). The points of intersection of the two curves in (c) represent the simultaneous solutions of (1) and (8) in the given case.

In Fig. 2, in (d) the modified cross-receptance Φ' of the machine is reproduced, so that by correlation of the solutions in (c) with Φ' corresponding values of r (indicating corresponding limit chip width b) may be determined. The maximum negative value of Φ' gives the actual limit of stability case.

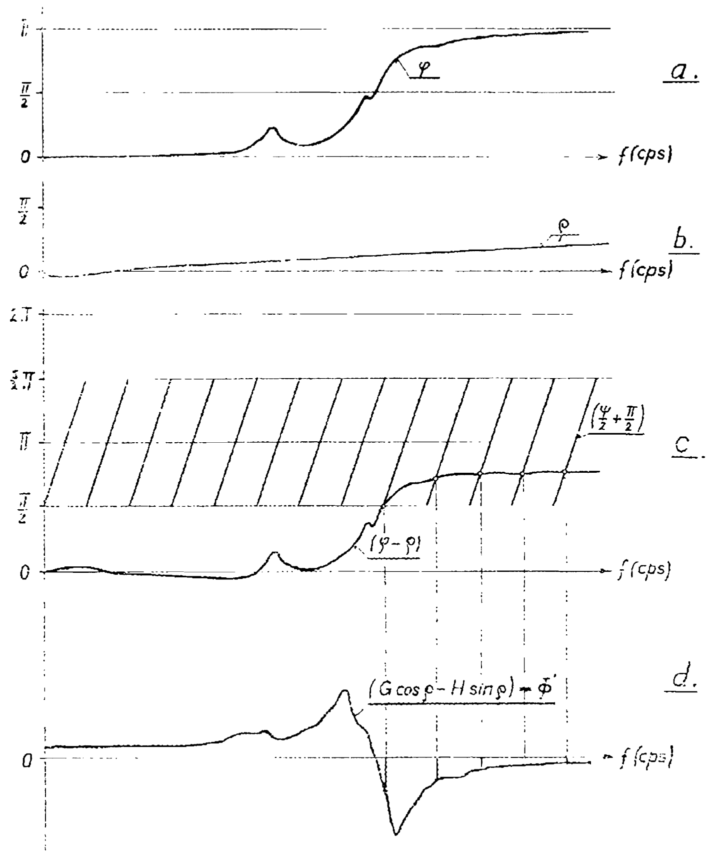

FIG. 2.

In the manner described above solutions for various speeds v may be easily obtained and stability charts of the type established by Tobias[(2)] may be constructed. For various values of v the slope of the lines $\left(\frac{\psi}{2}+\frac{\pi}{2}\right)$ will vary.

IRREGULAR PITCH OF CUTTER TEETH

The simplifying assumptions I and II are again made but the simplification III does not now apply. In this case, for a clarity of explanation, a real coupling coefficient, i.e. $\rho = 0$, is assumed. Principally, however, there would be no difficulty in taking a complex value for r. The irregularity in the tooth pitch is chosen as a periodic alternation of one greater and one smaller pitch.

Equations for forces on individual tools (see Fig. 3) may be written in an analogous manner to that of the previous case of the regular pitch:

$$\begin{aligned} P_1 &= -r_0\,(Y - Ye^{j\psi_1}) \\ P_2 &= -r_0\,(Y - Ye^{j\psi_2}) \\ P_3 &= -r_0\,(Y - Ye^{j\psi_1}) \text{ etc.} \end{aligned}$$

However, the phase shifts ψ_1 and ψ_2 are different now because of the different values of l to be inserted into the geometric condition (1):

$$\psi_1 = \frac{\omega\, l_1}{v}, \quad \psi_2 = \frac{\omega\, l_2}{v} \tag{10}$$

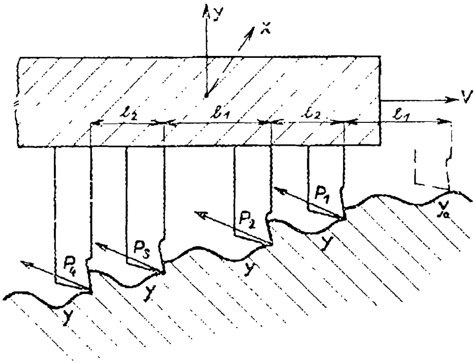

FIG. 3.

If it is assumed that an equal number of teeth of both pitches cut simultaneously, the total cutting force will be:

$$P = \sum_{1}^{n} P_1 = \tfrac{1}{2}\, r\, Y\,(2 - e^{j\psi_1} - e^{j\psi_2}) \tag{11}$$

Combining equations (3), (4), (11), the following expression is obtained

$$(e^{j\psi_1} + e^{j\psi_2} - 2) = \frac{2}{r\,F}\, e^{j\psi} \tag{12}$$

If the idea of a mean value ψ and its variation Δ is introduced:

$$\psi = \frac{\psi_1 + \psi_2}{2}, \qquad \Delta = \frac{|\psi_1 - \psi_2|}{2} \tag{13}$$

and

$$t = \frac{|l_1 - l_2|}{2} = \frac{v\Delta}{\omega} \tag{14}$$

where t is the irregularity of tooth pitches, then equation (12) may be transformed into:

$$(\cos \Delta \,.\, e^{j\psi} - 1) = \frac{1}{r\,F}\, e^{j\psi} \tag{15}$$

This vectorial equation is analogous to equation (7) for the case of regular pitch and is very important.

The frequencies of vibration which satisfy, simultaneously, equation (15) and the geometric conditions (10) can be obtained in a similar manner to that shown in Fig. 2 for regular pitches. These are obtained by a graphical method as intersection points of the arguments of both sides of equation (15), when they are plotted as functions of ω. Let us, for brevity, denote

$$(\cos \Delta\, e^{j\psi} - 1) = De^{j\delta} \tag{16}$$

Then the frequencies of chatter are found at the points, where

$$\delta = \psi \tag{17}$$

The graph of the argument ψ of the right-hand side of equation (15) is identical with that for the case of regular pitch, being the phase component of the cross-receptance.

The graph of the argument δ may be established for given particular values of r, l_1, l_2 by calculation, point by point, an example being given in Fig. 4a. The special form of this

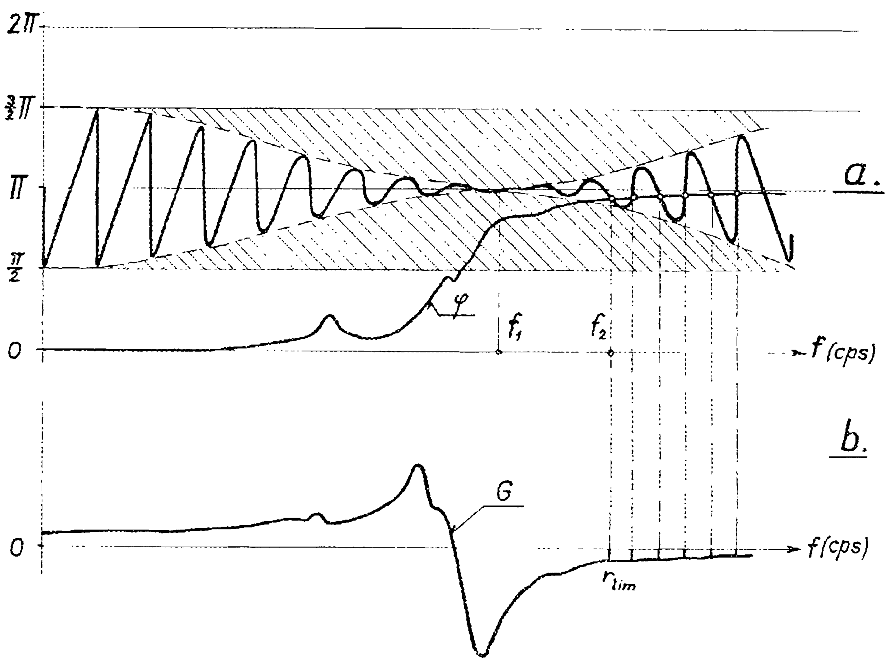

FIG. 4.

function has great significance for the stability of milling. From Fig. 4a it may be seen that in some frequency ranges δ differs somewhat from the analogous functions $\left(\frac{\psi}{2} + \frac{\pi}{2}\right)$ of the case of regular pitch (shown in 2cu). In these ranges it approaches the value $\delta = \pi$, as shown in Fig. 4a in the range around the frequency f_1. It should be realized that the form of $\delta(\omega)$ is such that in some frequency ranges it covers a wide range of phase angles whilst in others it has a very narrow phase angle width. In those ranges, where δ varies over a wide range possibilities of intersections $\delta = \psi$ always exist and where δ varies little it is easy to avoid the intersections $\delta = \psi$.

The value of the coupling coefficient r at the limit of stability may be derived from equation (15):

$$r = -\frac{1}{2G} \cdot \frac{1}{1 - \dfrac{\sin^2 \Delta}{2\,(1 - \cos \Delta \cos \psi)}} \tag{18}$$

Let

$$\tau = \frac{1}{1 - \dfrac{\sin^2 \Delta}{2\,(1 - \cos \Delta \cos \psi)}} \tag{18a}$$

then the value of τ is positive for all values of Δ and ψ and always $1 \leqslant \tau \leqslant 2$. The value of r may increase therefore up to double the value obtained in the case of regular pitch—equation (9a).

The maximum increase occurs just at the frequency f_1. However, because in the range around f_1, chatter can be avoided completely, as will be explained later, it is possible to neglect the increase of stability for values of $\tau > 1$ and it is suitable to take $\tau = 1$ and accept equation (9a) also for the case of irregular pitch.

Therefore, for frequencies ω established by the intersection points in Fig. 4a corresponding values of r may be found on the curve G of Fig. 4b. The actual chatter case will be that for which the value of r is a minimum, as shown by r_{lim} in Fig. 4b at the frequency f_e.

The middle f_1 of the frequency range, where δ varies in a narrow range occurs for the value of $\cos \Delta = 0$, i.e. for

$$\Delta = n \frac{\pi}{2}; \ (n = 1, 3, 5, \ldots) \tag{19}$$

By inserting equation (19) into (14) the frequency f_1 is derived as:

$$f_1 = n \cdot \frac{v}{4\,t} \tag{20}$$

The most important possibility of how to increase the stability of milling is evident from the preceding explanations. The range of frequencies, with the narrow band of values δ, should be located so that it coincides with the range where the G curve has greater negative values corresponding to small values of the limit r. Then it it easy to avoid intersections of the δ and ψ curves in this range and eliminate possible chatter cases with small r values as illustrated in the example Fig. 4. This recommended arrangement is achieved by the corresponding choice of the pitch irregularity t according to equation (20).

This method is limited, of course, to the application of a particular combination of a speed v and an irregularity t. The maximum effect is obtained in machines, where the real part G of the cross-receptance has an expressed negative maximum in a narrow frequency range.

It is obvious that this is a very suitable method for particular cases of difficult milling operations, mainly in series production. Again it should be understood that in the design of a universal machine tool intended for all kinds of various operations the geometric condition (1) cannot be utilized and the simple method as described in ref. 1 is fully justified.

TEST RESULTS

To verify the results of the theory described in the foregoing, experiments have been carried out on a vertical milling machine. A face milling cutter of diameter 315 mm with 16 cemented carbide tipped teeth has been used and very flexible hollow cast iron boxes were chosen as workpieces. The flexibility of the workpieces exceeded by far that of the machine. The limit of stability was measured by the limit depth of cut.

The results of one series of tests are plotted in Fig. 5. Firstly a regular pitch of teeth was used at different cutting speeds. The limit depth of cut was comparatively small and nearly equal for all speeds. The frequency of chatter was about 240 cps. Furthermore, three

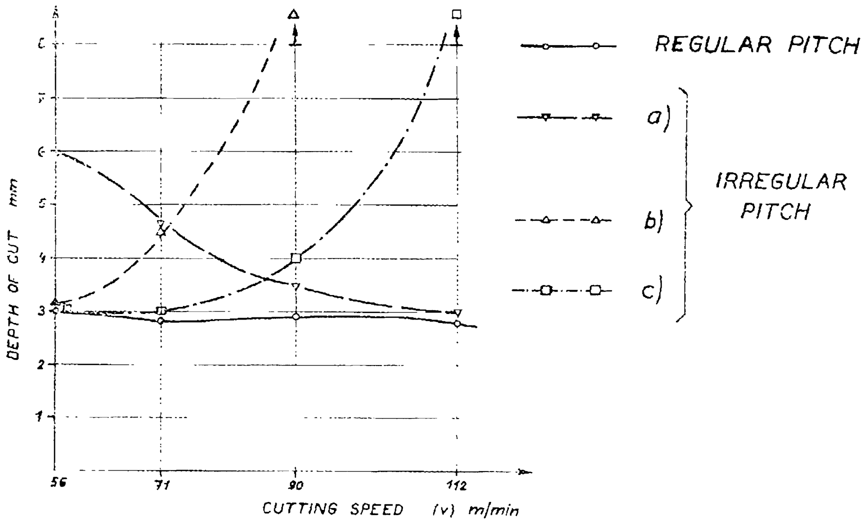

FIG. 5.

different pitch irregularities were tested. They were chosen so as to have the maximum effect for speeds of 56, 90 and 112 m/min at the frequency of 240 cps. The optimum pitch irregularities for the three individual speeds are:

(a) for $v = 56$ m/min $t_1 = 1$ mm (see equation (16))

(b) for $v = 90$ m/min $t_2 = 1{\cdot}5$ mm

(c) for $v = 112$ m/min $t_3 = 2$ mm

It can be seen from the diagram that the increase in stability was remarkable. At the same time a shift of chatter frequency was also observed.

From these results it is obvious, that the agreement with the conclusions of the theory is good. Therefore, the use of milling cutters of irregular pitch can be accepted as an important method by which the stability of milling machines can be increased.

REFERENCES

1. TLUSTÝ, J., A method of analysis of machine tool stability, MTDR Conference, Manchester, 1965.
2. TOBIAS, S. A., The vibrations of vertical milling machines under test and working condition, *Proc. J. Mech. Eng.*, Vol. 173, 1959.
3. TOBIAS, S. A. and SMITH, J. D., The dynamic cutting of metals, *Int. J. MTDR*, Vol. 1, No. 4, 1961.
4. LEMON, J. R. and LONG, G. W., Survey of chatter research at the Cincinnati milling machine company, MTDR Conference, Birmingham, 1964.